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2-D Wavelet Transform for Image Smoothing and Edge Detection

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ABSTRACT

The Wavelet Transform (WT) has gained widespread acceptance ranging from time dependent signal processing to image processing because of their inherent multi-resolution nature. In this paper, the two-dimensional wavelet transform is used in smoothing and detection of edges of gray-scale images. Elimination of high-frequency components by zeroing details gives smoother image and elimination of low frequency components, i.e. zeroing approximation gives the edges is verified here. The transmission and reconstruction of four-scale wavelet transmission is also analyzed in this paper.

Keywords: STFT, de-noising of signal, basis function, CWT and DWT tree.

INTRODUCTION

In recent years, for signal and image processing wavelet transform is given more importance than short time Fourier transform (STFT). Wavelet is an oscillatory function of finite duration. Wavelet transform maps any signal f(t) in time domain to a function W(a, b) of two continuous real variables a and b responsible for wavelet dilation and translation respectively [1, 2]. Discrete wavelet transformation (DWT) also transforms a continuous time signal f(t) but the discretization is done only in a and b variables is applied for image processing of this paper.

A wavelet is a waveform of effectively limited duration that has an average value of zero. A sinusoidal wave $y(t)=A\cos\omega_c t$ is continuous over the interval $[\infty, -\infty]$ and the energy signal is infinite over the interval $[\infty, -\infty]$. The energy of $y(t)$ is uniformly distributed over the entire interval. If the wave y(t) is modulated by a smooth Gaussian window function g(t) = e^{-t^2}, the modulated wave known as Morlet wavelet, $\psi(t) = g(t)y(t) = Ae^{-t^2}\cos(\omega_c t)$ is a wavelet of infinite duration and is also continuous

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over the interval $[\infty, -\infty]$. But almost all the energy in this wavelet is confined within a finite interval.

For example when $\omega_c = \pi\sqrt{\frac{2}{\ln 2}}$, more than 99% of total energy of $\psi(t)$ lies in the interval $|t| \leq 2.5$ sec and such wave $\psi(t)$ is known as wavelet. The real-value Morlet wavelet is $\psi(t) = e^{-t^2}\cos\left(\pi\sqrt{\frac{2}{\ln 2}}\right)$.

Wavelet function Ψ(t) has two main properties [3]:

- $\int_{-\infty}^{0} \Psi(t)dt = 0;$ This means, the function is oscillatory or has wavy appearance.
- $\int_{-\infty}^{0} |\Psi(t)|^2 dt < \infty;$ This implies that most of the energy in Ψ(t) is confined to a finite duration.

Wavelet is expressed in another form like,

$\psi_{a,b} = \psi\left(\frac{t-b}{a}\right)dt$; where a and b are the scaling and shifting parameters.

Scaling a wavelet simply means stretching (or compressing) it. The smaller the scale factor, the more "compressed" the wavelet. Shifting a wavelet simply means delaying the function [4].

For example, the wavelet $\psi(t) = e^{-t^2}\cos\left(\pi\sqrt{\frac{2}{\ln 2}}t\right)$ is known as Morlet wavelet and $\psi(t) = (1-2t^2)e^{-t^2}$ is known as Maxican hat wavelet.

Let f(t) be any square integrable function. The CWT or continuous-time wavelet transform of f(t) with respect to a wavelet $\psi(t)$ is defined as,

$$W(a,b) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{|a|}}\psi^*\left(\frac{t-b}{a}\right)dt, \quad (1)$$

where a and b are real and * denotes conjugation.

Equation (1) can be written in a more compact form by defining,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \quad (2)$$

Combining equation (1) and equation (2),

$$W(a,b) = \int_{-\infty}^{\infty} f(t)\psi_{a,b}(t)dt$$

One-dimensional wavelet transform is usually used in decomposition and synthesis of voice or data signal but for image analysis two-dimensional wavelet transform is used which is much more complicated than that of one-dimensional case. The DWT is computed by successive low pass and high pass filtering of the discrete time-domain signal [5]. In this paper section 2 deals with complete theoretical analysis of two-dimensional wavelet along with its decomposition and synthesis technique, section 3 gives the smoothen image and edges of a gray scale image with variation of details and approximation part of wavelet transformed image and finally section 4 concludes the entire analysis.

METHODOLOGY

The most important part of any signal is the low frequency content which gives the signal identity and the high frequency component on the other hand only changes the tone of a signal. When a signal is simultaneously passed through a low pass and high pass filter, the output of the low pass filter contains the low frequency content of the signal is called approximation, gives the signal identity. The output of the high pass signal is called details provides the tone of the signal [3, 6].
For digital sequence the number of samples at the output of both low pass and high pass filter is equal to the number of samples of original message. Therefore, for synthesis of the sequences both approximation and details are down sampled like Fig. 1.

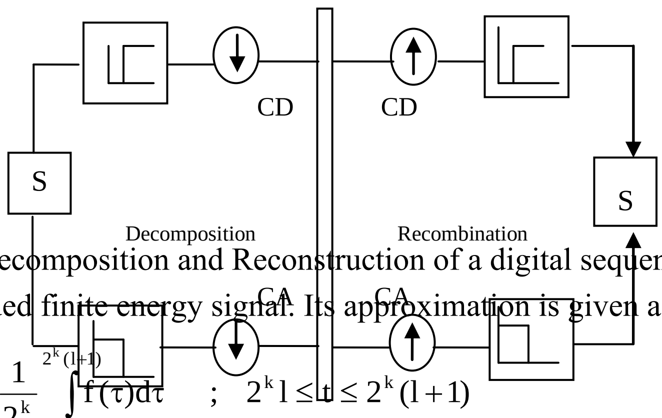


Fig. 1 Decomposition and Reconstruction of a digital sequence.

Let $f(t)$ is a real-valued finite energy signal. Its approximation is given as:

$$f(t) = \frac{1}{2^k} \int_{2^k l}^{2^k (l+1)} f(\tau) d\tau \quad ; \quad 2^k l \le t \le 2^k (l+1)$$

(3)

$f(t)$ is also written as a linear combination of basis function $\phi(2^{-k} t)$ and approximation coefficient $c(k,l)$ like,

$$f_k(t) = \sum_{l=-\infty}^{\infty} c(k,l)\ \phi\ 2^{-k}(t-l)$$

(4)

Where, $\phi\ 2^{-k}(t-l) = \Pi\left(\frac{t - l2^k - \frac{1}{2}}{2^k}\right)$ and

$$c(k,l) = \frac{1}{2^k}\int_{2^k l}^{2^k(l+1)} f(t)dt$$

Similarly the details function is,

$$g_k(t) = \sum_{l=-\infty}^{\infty} d(k,l)\ \phi\ 2^{-k}(t-l) \qquad (5)$$

Where, the details coefficient,

$$d(k,l) = \frac{1}{2^k}\int_{2^k l}^{2^k(l+1)} f(t)\phi(2^{-k}t - l)dt$$

and $\phi(t)$ is the basis function.

Here both $f_k(t)$ and $g_k(t)$ lies in linear vector space V_k and W_k respectively. In DWT tree approach, only approximation is decomposed on every step like Fig. 2.

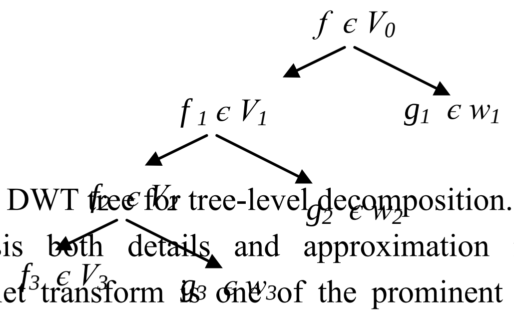


Fig. 2 DWT tree for tree-level decomposition.

In wavelet packet analysis both details and approximation is decomposed.Two-dimensional discrete wavelet transform is one of the prominent mathematical tool to resolve components of an image for analysis and modification [7, 8, 9].

Let us consider a square and integrable function f(x,y) can be expressed as a linear combination of three two-dimensional wavelets – namely, $S_{\Phi\Psi}$ (x, y), $S_{\Psi\Phi}$ (x, y) and $S_{\Psi\Psi}$ (x, y) like [10, 11],

$$f(x,y) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} b_k(n,p)s_{\phi_\psi}(2^{-k}x - n, 2^{-l}y - p) +$$

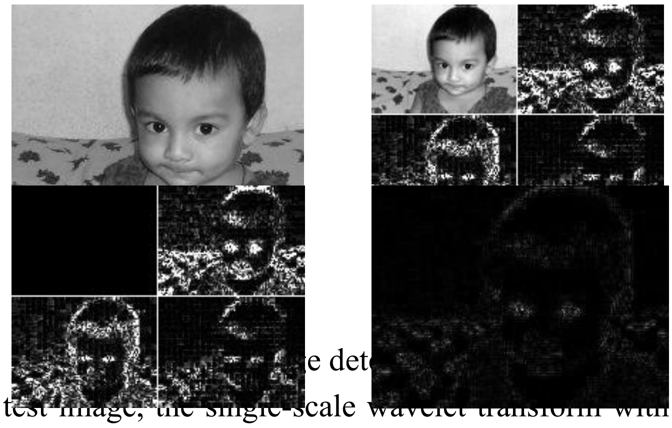
$$\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} c_k(n,p)s_{\psi_\phi}(2^{-k}x - n, 2^{-l}y - p) + \qquad (6)$$

$$\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} d_k(n,p)s_{\psi_\psi}(2^{-k}x - n, 2^{-l}y - p),$$

where, the coefficient $b_k(n, p)$, $c_k(n, p)$ and $\mathrm{d_k(n,p)}$ can be evaluated based on analysis of reference of book.

RESULTS AND DISCUSSIONS

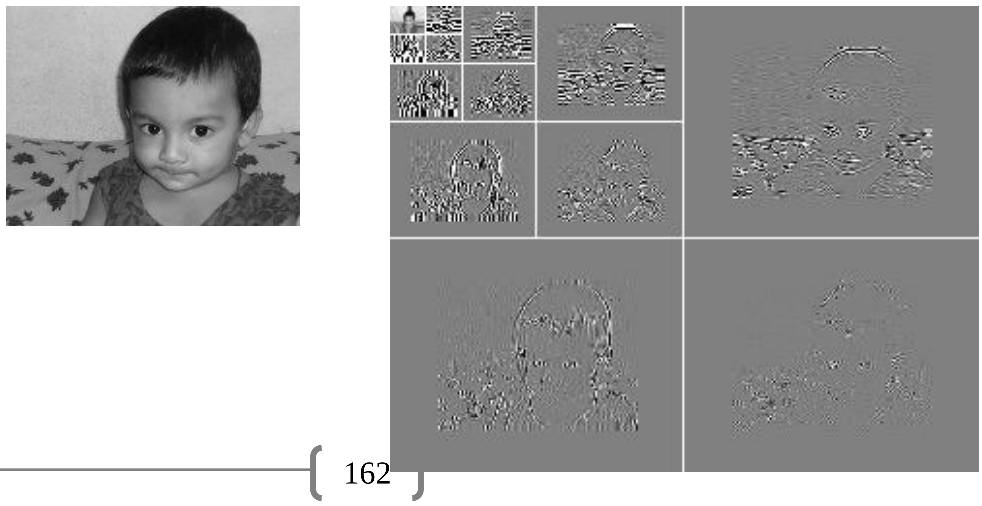
In this paper, a 200 Ḥ 150 test image is considered in Fig. 3(a). This image is used here to illustrate smoothing and edge detection with two dimensional (2-D) wavelet transform. Fig. 3 demonstrates the use of 2-D wavelet transform in edge detection.



e det

Here, Fig (a) is a simple test image, the single-scale wavelet transform with respect to 4th order symlet wavelet (absolute values scaled by 6) of the test image is performed in Fig (b). Fig (c) represents the modified transform by zeroing all approximation coefficients (absolute values scaled by 6); and Fig (d) shows the edge image resulting from computing the absolute value of the inverse transform.

The horizontal, vertical, and diagonal directionality of the single-scale wavelet transform of Fig. 3(a) with respect to 'symlet' wavelet is clearly visible in Fig. 3(b). Note, for example, that the horizontal edges of the original image are present in the horizontal detail coefficients of the upper-right quadrant of Fig. 3(b). The vertical edges of the image can be similarly identified in the vertical detail coefficients of the lower-left quadrant. To become this information into a single edge image, we simply zero the approximation coefficients of the generated transform, compute its inverse, and take the absolute value. The modified transform and resulting edge image are shown in Figs. 3(c) and 3(d), respectively.



(b)

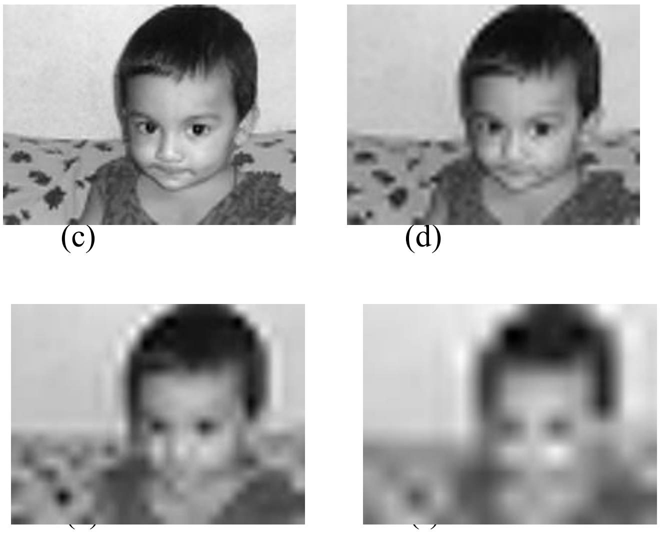
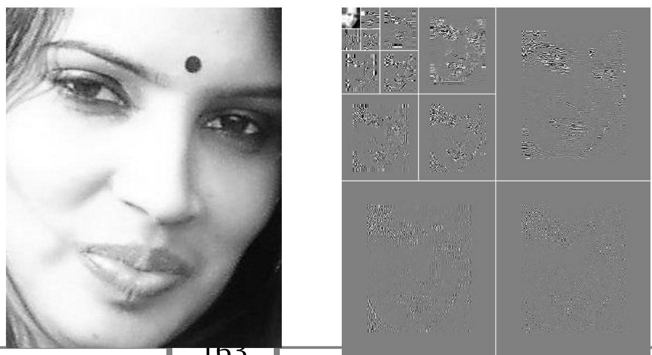


Fig. 4 Wavelet-based image smoothing

Here, Fig (a) is a simple test image and Fig (b) is the four-order wavelet transform with respect to symlet wavelet (scaled by 20) of the test image. Fig (c) represents the inverse transform modified by zeroing first-level detail coefficients and Fig (d), (e) and (f) shows the inverse transform modified by zeroing second-level detail coefficients, third-level detail coefficients and fourth-level detail coefficients respectively.

Wavelets are also effective instruments for smoothing and blurring images. Fig. 4 illustrates smoothing application of with 2-D wavelet transform. The test image of Fig. 3(a) is considered again in Fig. 4(a) for this purpose. Its wavelet transform with respect to fourth-order symlets is shown in Fig. 4(b), where it is clear that a four-scale decomposition has been performed. Note that the smoothed image in Fig. 4(c) is only slightly blared, as it was obtained by zeroing only the first-level detail coefficients of the original image's wavelet transform. Additional blaring is present in the second result-Fig. 4(d)-which shows the effect of zeroing the second level detail coefficients as well. The coefficient zeroing process continues in Fig. 4(e), where the third level of details is zeroed, and concludes with Fig. 4(f), where all the detail coefficients have been eliminated.

The transmission and reconstruction of four-scale wavelet transform is demonstrated in Fig. 5. This example uses a typical gray level test image of 250 Ḥ 312 pixels shown in Fig. 5(a).



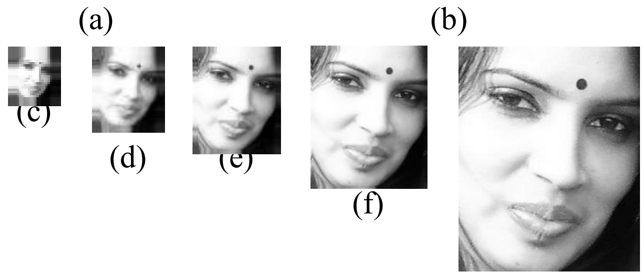


Fig. 5 Progressive reconstruction: (a) A simple test image; (b) Its four-order wavelet transform (scaled by 8); (c) the fourth-level approximation image from the upper-left corner; (d) a refined approximation incorporating the fourth level details; (e) through (g) further resolution improvements incorporating higher level details.

Fig. 5(a) shows a simple test image for reconstruction of the image. then the fourth-order wavelet transform is performing on the image of Fig. 5(a) by the scaling factor 8 and the resultant image is depict in fig. 5(b). Fig. 5(c) represents the fourth-level approximation image from the upper-left corner. and Fig. 5(d) shows a refined approximation incorporating the fourth level details. from Fig. 5(e) through Fig. 5(g) shows the further resolution improvements incorporating the higher level details.

CONCLUSION

The paper depicts the impact of noise on transformed parameters of wavelet for both one and two dimensional signals including the remedial measure. In this paper, a set of MATLAB based user defined functions of 2-D wavelet transform is developed by the authors in their own way. The functions have the flexibility of manipulating the quality of each pixel by simply changing the parameters of the program. All the images considered here in gray scale for simplicity of analysis and the yields logical result. Still there is a scope of the paper to do the same job for colour images by simply increasing the dimension of each element of the pixel matrix.

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